Temporal dynamics of Kerr frequency combs in whispering-gallery mode resonators

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Kerr frequency combs in whispering-gallery mode resonators:

- Sets of regularly spaced spectral lines in the optic frequency range
- Rely on four-wave-mixing; needs high Q-factor resonator \( \sim 10^9 \)
- Various applications: time-frequency metrology, spectroscopy, navigation systems, sensing, ...
Motivation:

- Kerr combs are obtained by a few group in the world
- **Understand** the mechanisms
- **Improve combs**: stability, frequency interval, …
- A few models \(^1\)

\(^1\)A. A. Savchenkov *et al*, arXiv:1111.3922v1 (2011)
Outline

Introduction & Motivation

Experimental Kerr combs
  Experimental Setup
  Experimental Combs

Theoretical model
  Theory
  Numerical Simulations
  Time evolution
  Phase locking dynamics

Conclusion & Perspectives
Experimental Setup

- MgF$_2$ polished disk, $5 \times 10^8$ Q-factor
- Evanescent coupling with a tapered fiber
- Monitoring optical spectrum and transmission
- $\lambda = 1550\text{nm}$, input power $\sim 100\text{mW}$
Experimental Setup

Green light coupling
Experimental Results

$\Delta f = 7 \text{ FSR}$

Multiple-FSR Combs: Primary comb
Experimental Results

Multiple-FSR Combs: Primary comb
**Experimental Results**

Multiple-FSR Combs: **Primary comb**

\[ \Delta f = 88 \text{ FSR} \]
Theoretical model

- Envelope approximation
- Modal description
- Dispersion and Kerr nonlinearity

\[ \dot{A}_\eta = -\frac{1}{2} \Delta \omega_\eta A_\eta - i g_0 \sum_{\alpha, \beta, \mu} \Lambda_{\alpha \beta \mu \eta} A_\alpha A_\beta^* A_\mu e^{i \varpi_{\alpha \beta \mu \eta} t} \]

\[ + \frac{1}{2} \Delta \omega_\eta F_\eta e^{i (\Omega_0 - \omega_\eta) t} \]

Theoretical model\(^2\)

- Envelope approximation
- Modal description
- Dispersion and Kerr nonlinearity

\[
\dot{A}_\eta = \ldots
\]

Theoretical model

- Envelope approximation
- Modal description
- Dispersion and Kerr nonlinearity

\[ \dot{A}_\eta = - \frac{1}{2} \Delta \omega_\eta A_\eta \]

Resonator bandwidth

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Theoretical model

- Envelope approximation
- Modal description
- Dispersion and Kerr nonlinearity

\[
\dot{A}_\eta = -\frac{1}{2} \Delta \omega_\eta A_\eta \\
- ig_0 \sum_{\alpha,\beta,\mu} \Lambda^{\alpha \beta \mu}_\eta A_\alpha A^*_\beta A_\mu e^{i\omega_{\alpha \beta \mu \eta} t}
\]

Resonator bandwidth

Four-wave mixing

Theoretical model

- Envelope approximation
- Modal description
- Dispersion and Kerr nonlinearity

\[
\dot{A}_\eta = -\frac{1}{2} \Delta \omega_\eta A_\eta
- ig_0 \sum_{\alpha, \beta, \mu} \Lambda^\alpha_\eta A_\alpha A^*_\beta A_\mu e^{i\omega_{\alpha\beta\mu_\eta} t}
+ \frac{1}{2} \Delta \omega_\eta F_\eta e^{i(\Omega_0 - \omega_\eta) t}
\]

Resonator bandwidth
Four-wave mixing
External excitation

Stability diagram

- Combs threshold: $|A_0|_\text{th}^2 = \Delta \omega_0 / g_0$
- Given detuning $\sigma = \Omega_0 - \omega_0$
Combs threshold: $|A_0|^2_{\text{th}} = \Delta \omega_0 / g_0$

Given detuning $\sigma = \Omega_0 - \omega_0$

$\sigma = 1.5 \Delta \omega_0$
Combs threshold: $|A_0|_{th}^2 = \Delta \omega_0 / g_0$

Given detuning $\sigma = \Omega_0 - \omega_0$

$\sigma = 4\Delta \omega_0$
Numerical Simulations

$|A_0|^2 = 1.01|A_0|^2_{th}$
Numerical Simulations

\[ |A_0|^2 = 1.01|A_0|_{th} \]

\[ \Delta f = 9 \text{ FSR} \]

\[ \eta = 1.11 \]
Numerical Simulations

\[ |A_0|^2 = 1.01|A_0|^2_{\text{th}} \]

\[ \Delta f = 9 \text{ FSR} \]

\[ \text{Mode number } \eta \]

\[ P \text{ [dB]} \]

\[ P \text{ [dB]} \]

Mode number η
Numerical Simulations

\[ |A_0|^2 = 1.01|A_0|^2_{\text{th}} \]

\[ \Delta f = 9 \text{ FSR} \]

\[ 1.11 \]

\[ 1.63 \]

\[ 1.96 \]
Focus on the first mode of the primary comb
Time evolution: low pump power

Focus on the first mode of the primary comb

![Graph showing time evolution of different modes with various power levels](image-url)
**Time evolution: high pump power**

- Stable comb to oscillations and chaos
Phase locking dynamics: primary comb

- Evolution of the relative phase of the modes
- Dispersion corrected: \( \arg\left( A_\eta e^{\omega_{\eta000}t}/A_0 \right) \)
Phase locking dynamics: primary comb

- Evolution of the relative phase of the modes
- Dispersion corrected: \[ \arg(A_\eta e^{\omega_\eta t_0 t}/A_0) \]
- Low excitation, primary comb only
Phase locking dynamics: secondary comb

- Phase-locking still occurs in the primary comb
- Secondary comb is not in phase with the pump
Phase locking dynamics: chaotic comb

- Phase-locking still occurs in the primary comb
- Chaotic behavior
Conclusion & Perspectives

- Experimental generation of multiple-FSR Kerr combs
Experimental generation of multiple-FSR Kerr combs

Theoretical model: modal description
Conclusion & Perspectives

- Experimental generation of multiple-FSR Kerr combs
- Theoretical model: modal description
- Numerical simulations to study the transient regime:
  - Stable combs at low excitation
  - Oscillation regimes with secondary comb
  - Chaotic behavior with each mode of the comb populated
Conclusion & Perspectives

- Experimental generation of multiple-FSR Kerr combs
- Theoretical model: modal description
- Numerical simulations to study the transient regime:
  - Stable combs at low excitation
  - Oscillation regimes with secondary comb
  - Chaotic behavior with each mode of the comb populated
- Numerical study of the phase locking phenomena:
  - Primary comb always phase-locked
  - Secondary comb does not follow the same relationship
  - Chaotic behavior at higher excitation level
Perspectives

- Understanding phase-locking phenomenon
- Finding best regime for the generation of ultra-pure microwave
- Evaluate the quality of the phase locking
- Timescale of phase locking?
Thanks for your attention!